UGEB2530 Game and strategic thinking
Solution to Assignment 5

1. Find the threat solutions, that is find the threat strategy and the payoff of each player, of the games with the following bimatrices. **Solution:**

   (a) The threat strategy for player I is $(0.3, 0.7)$ and the prudential strategy for player I is $(0.6, 0.4)$. So the threat differential is:

   $$\begin{bmatrix} 0.3 & 0,7 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} = 2.2.$$

   Let x, y be the payoff of the player I and iI then:
   $x + y = 6$,
   $x - y = 2.2$
   we have $x = 4.1$, $y = 1.9$.

   (b) The threat strategy for player I is $(0.2, 0.8)$ and the prudential strategy for player I is $(0.6, 0.4)$. So the payoff of each player using the strategy are:

   $$v_I = \begin{bmatrix} 0.2 & 0,8 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} = 0.4.$$

   Let x, y be the payoff of the player I and iI then:
   $x + y = 8$,
   $x - y = 0.4$
   we have $x = 4.2$, $y = 3.8$.

2. **Solution:**
   We have John has to pay=$50 + \frac{100-(50+80)}{2} = 35$.
   And Peter has to pay=$80 + \frac{100-(50+80)}{2} = 65$.

3. **Solution:**

   (a)

$Coalition$	$v(s)$
$\{A\}$	0
$\{B\}$	0
$\{C\}$	0
$\{A, B\}$	3
$\{A, C\}$	5
$\{B, C\}$	2
$\{A, B, C\}$	6

   (b) Shapley?s value of A=$\frac{8}{3}$.
   Shapley?s value of B=$\frac{7}{6}$.
   Shapley?s value of C=$\frac{13}{6}$.

   (c) Town A should pay $11 - \frac{8}{3} = \frac{25}{3}$
   Town B should pay $7 - \frac{7}{6} = \frac{35}{6}$
   Town C should pay $8 - \frac{13}{6} = \frac{35}{6}$